

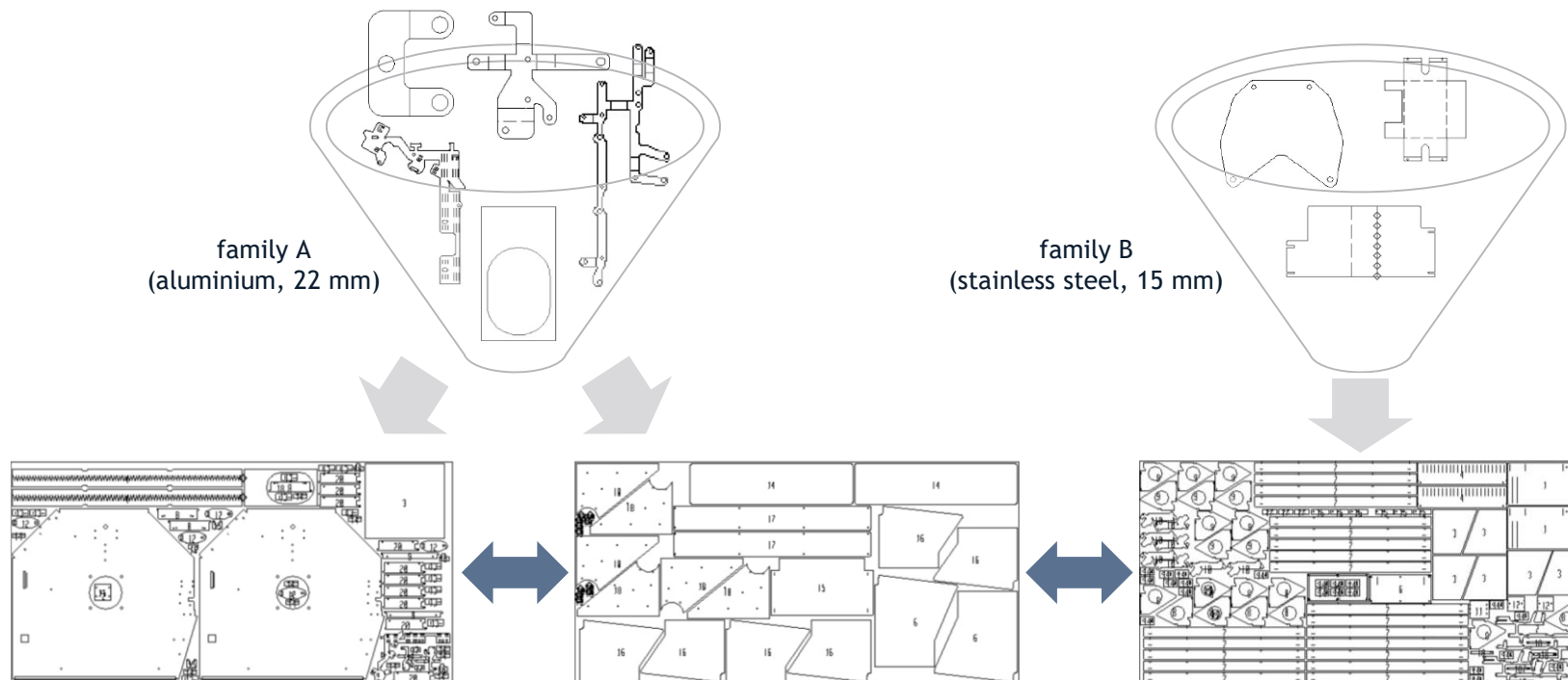
Approximate anticipation of base-level reactions  
by machine learning techniques used to substitute  
the solving of complex nesting problems

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Christian Gahm, **Aykut Uzunoglu**, Stefan Wahl,  
Chantal Ganschinietz, Axel Tuma  
University of Augsburg

# Motivation

- Application case: serial-batch scheduling in the metal-processing industry
  - Laser cutting



Decisions:

1. Batching
  - 1.1 Capacity-Checking
  - ML-Approximation**
2. Scheduling

- Simple approximation methods
- Hierarchical integration of the approximate anticipation by machine learning
- Prediction framework
  - Instance generation
  - Feature engineering
- Results

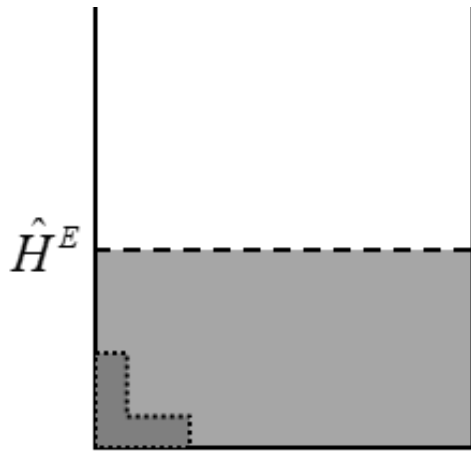
# Simple approximation methods

$$\hat{H}^E = \max\{\max_{j \in P}\{h_j\}, \sum_{j \in P} a_j^E / W\}$$

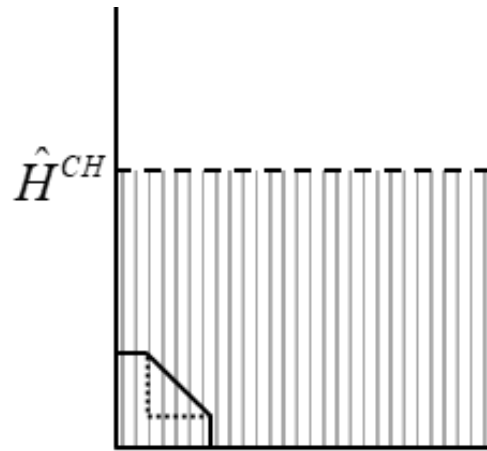
$$\hat{H}^{CH} = \max\{\max_{j \in P}\{h_j\}, \sum_{j \in P} a_j^{CH} / W\}$$

$$\hat{H}^{MBR} = \max\{\max_{j \in P}\{h_j\}, \sum_{j \in P} a_j^{MBR} / W\}$$

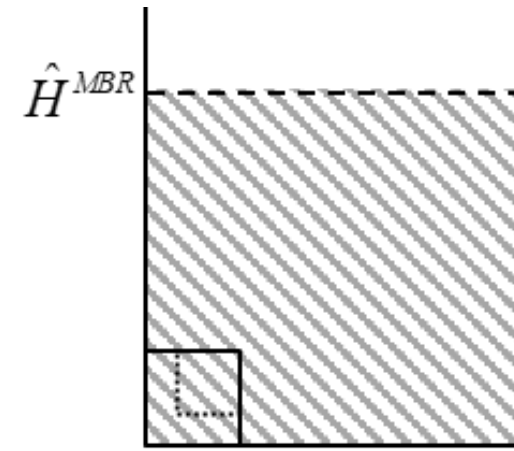
15 x 



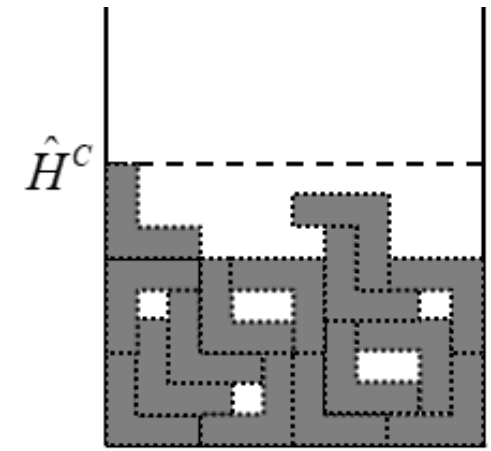
a) Optimistic (SA-E)



b) Conservative I (SA-CH)

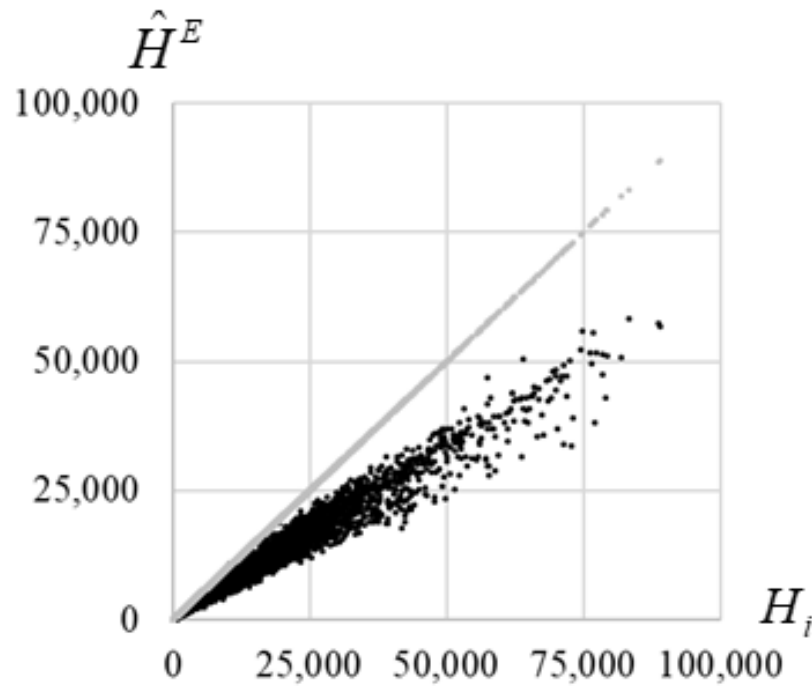


c) Conservative II (SA-MBR)

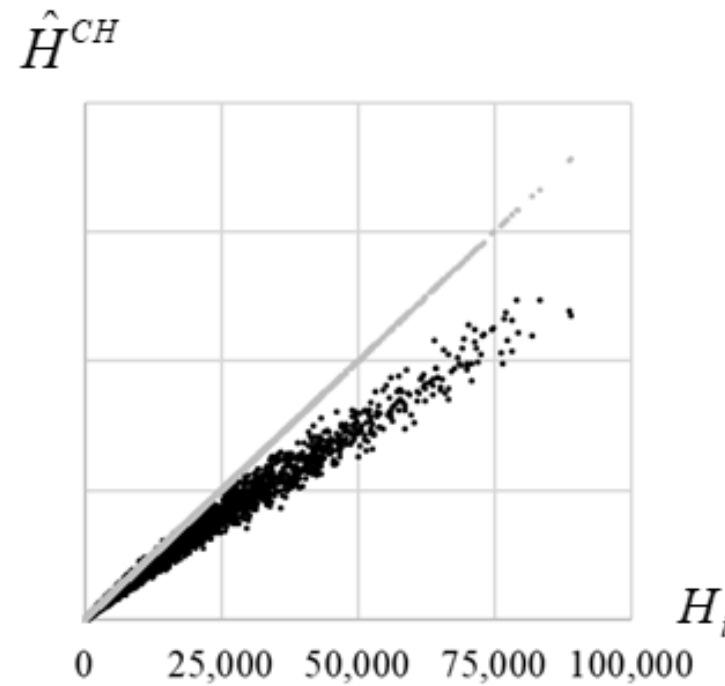


d) computed

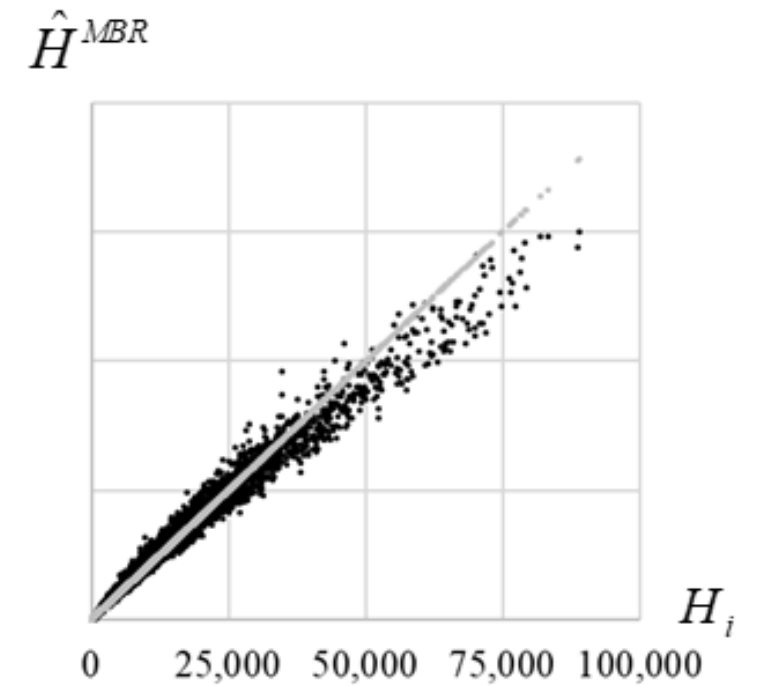
# Simple approximation methods



a) Optimistic  
(SA-E)

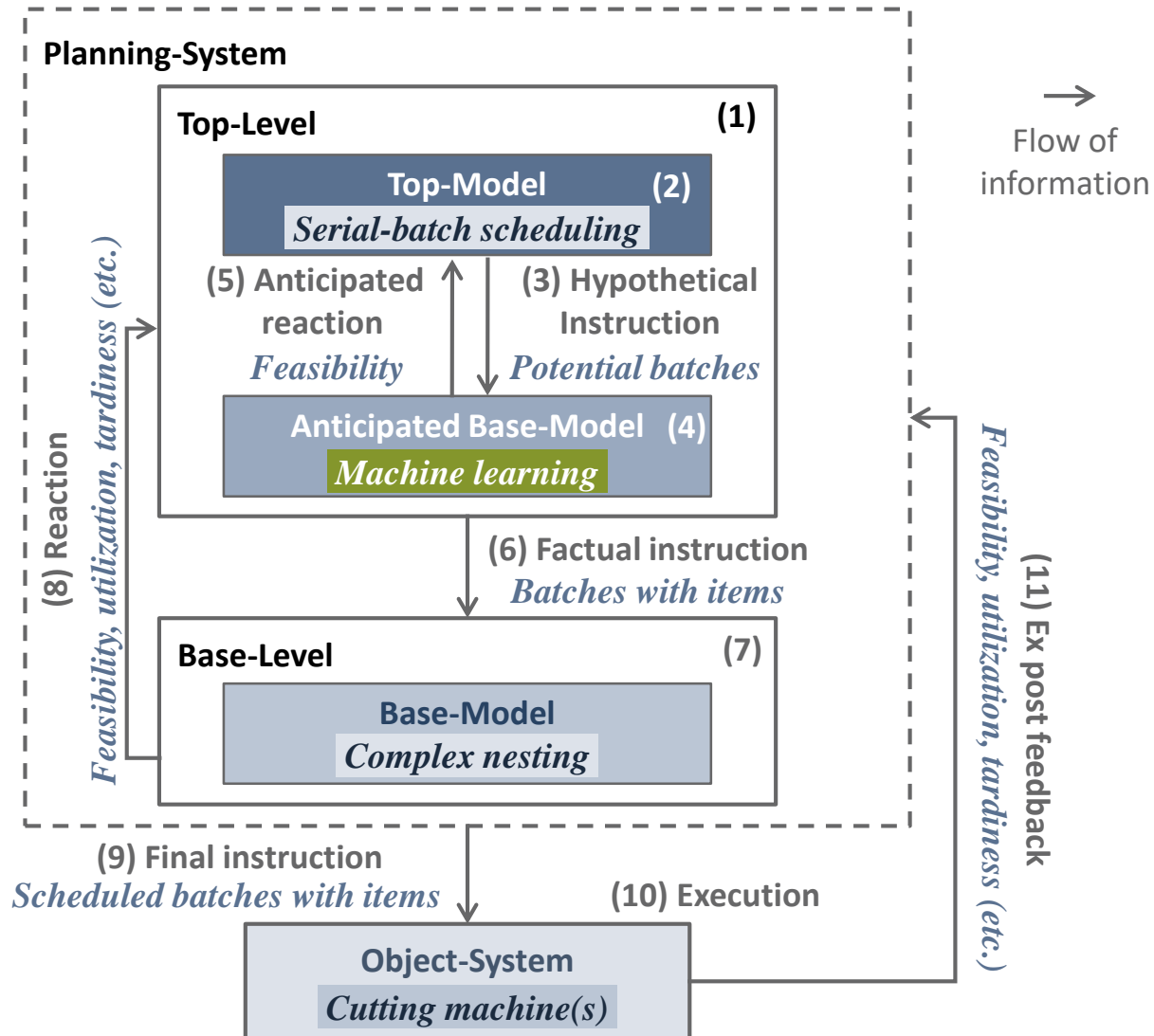


b) Conservative I  
(SA-CH)



c) Conservative II  
(SA-MBR)

# Hierarchical integration of the approximate anticipation by machine learning



## Serial-batch scheduling:

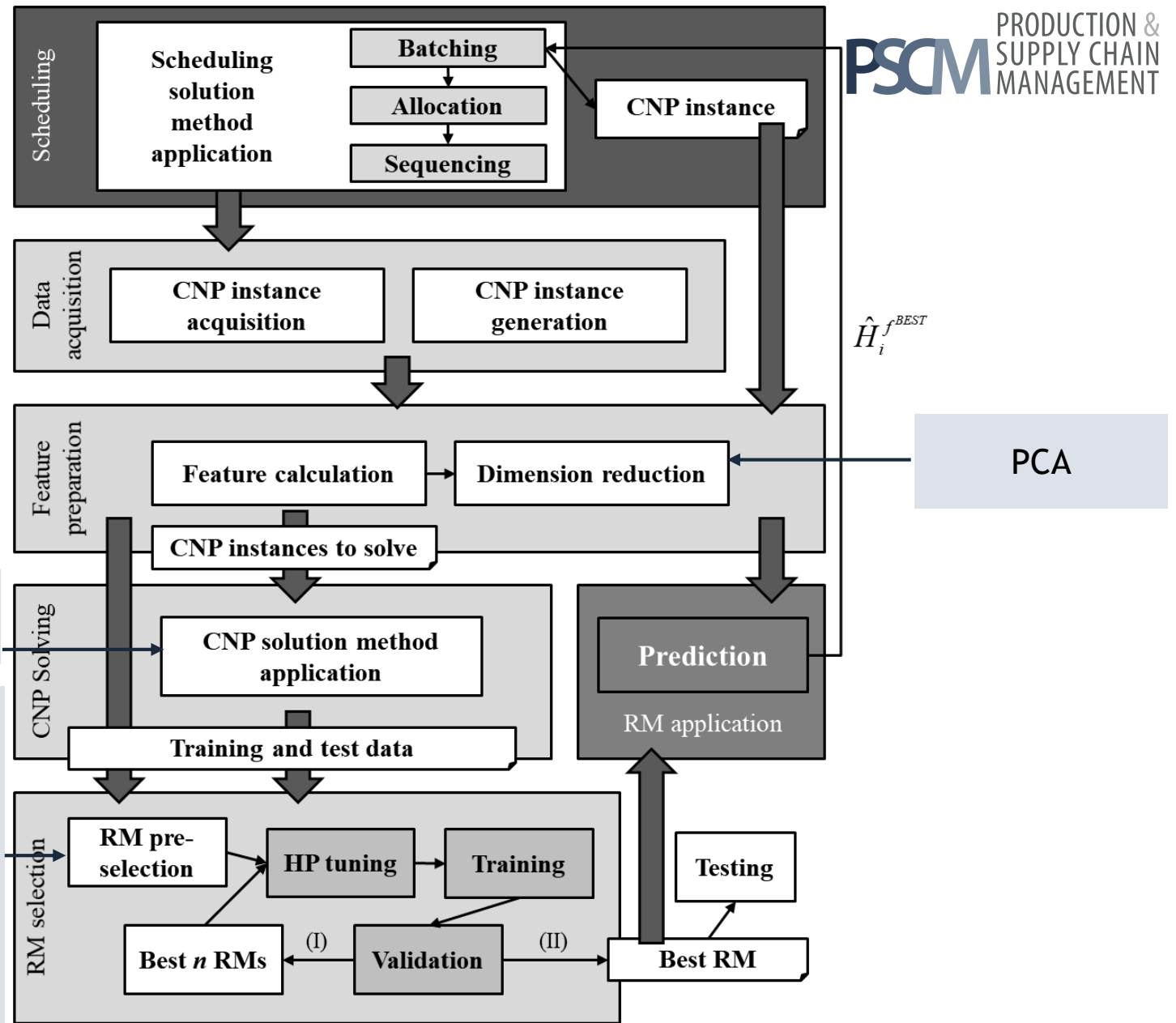
- Batching
- Allocation
- Sequencing

## Complex nesting:

- Two-dimensional strip packing problem
- Highly irregular shapes (concave with holes)
- Free rotations

# Prediction Framework

DeepNest



- Linear models, e.g., Ridge regression, Elastic net
- Linear models with polynomial features
- Neighborhood and kernel-based models, e.g., k-nearest neighbors regression, Support Vector Regression
- Decision tree based ensemble methods, e.g., Extremely randomized trees, Bagging regression trees, (Stochastic) Gradient boosted decision trees
- (Deep) Neural networks

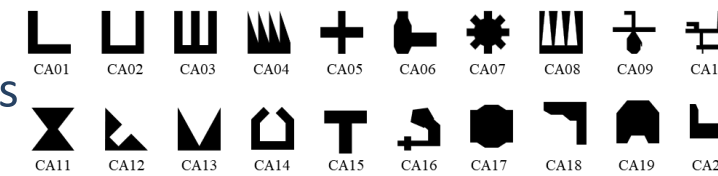
# Instance Generation

- Generation of complex items based on
  - 50 (10+20+20) elementary items
  - 4 scaling variations in width, 3 in height → 12 scaling variations
  - Every scaling variation is perturbed 10 times → 120 scaling variations
  - In total: 50 \* 120 = 6,000 items

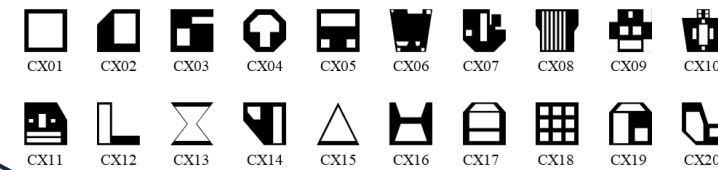
a) Convex items



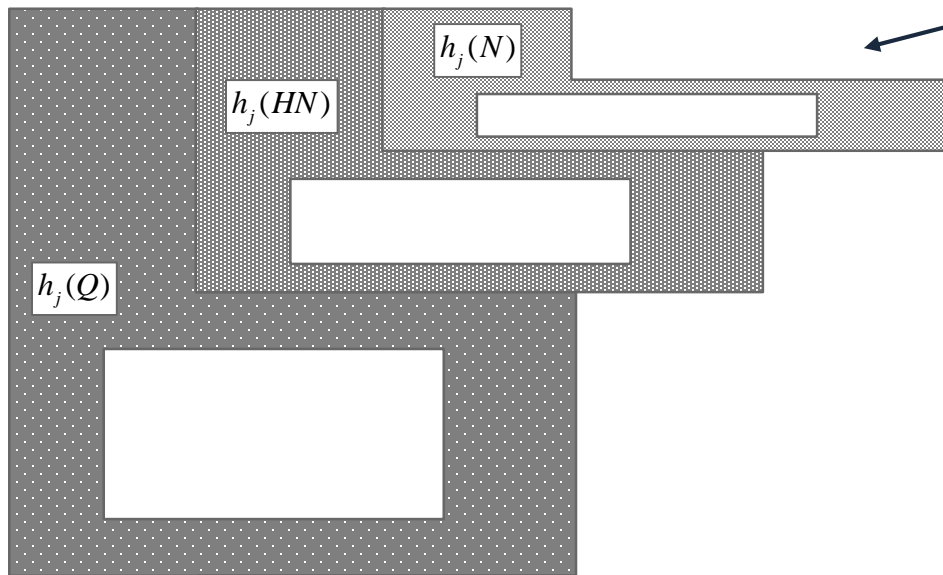
b) Concave items



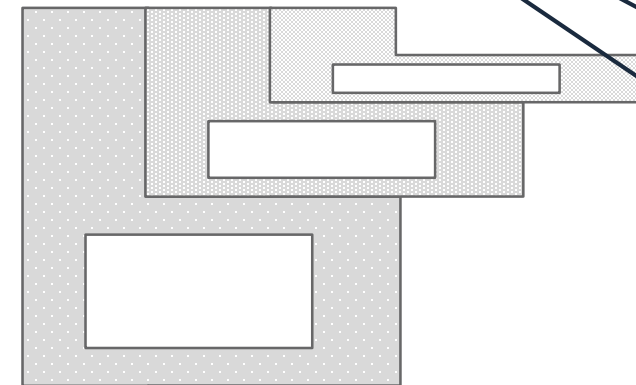
c) Complex items



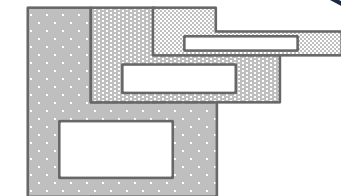
$w_j(XL)$



$w_j(L)$



$w_j(M)$



$w_j(S)$





# Instance Generation

createInstances( $N_c$  := number of instances per class,  $lb^n$ ,  $ub^n$ ,  $shapeRepository$ [ ])

For each  $OW \in \{SW, MW, LW\}$

For each  $ITA \in \{CV, CA, CX, CV+CA, CV+CX, CA+CX, CV+CA+CX\}$

For each  $ITH \in \{WH, SH\}$

For each  $IWA \in \{S+M, M+L, L+XL, S+M+L, M+L+XL, S+M+L+XL\}$

For each  $IHA \in \{Q, HN, N, Q+HN, Q+N, HN+N, Q+HN+N\}$

For  $i = 1$  to  $N_c = 50$

**// for each of the 1,764 instance classes; in total: 88,200 CNP instances**

$\beta_{ITA}^R := \sim U(BD)$ ;  $\beta_{IWA}^R := \sim U(BD)$ ;  $\beta_{IHA}^R := \sim U(BD)$ ;

$attPerm_{ITA} := getPerm(ITA)$ ;  $attPerm_{IWA} := getPerm(IWA)$ ;  $attP$

$S[ ] := getShapeSubsets(shapeRepository[ ], ITA, ITH, IWA, IHA)$

$n := \sim U(lb^n = 50, ub^n = 150)$

For  $j = 1$  to  $n$

$type_j := getTypeAttribute(\beta_{ITA}^R, attPerm_{ITA}, ITA)$ ; **// e.g**

$w_j := getWidthAttribute(\beta_{IWA}^R, attPerm_{IWA}, IWA)$ ; **// e.g**

$h_j := getHeightAttribute(\beta_{IHA}^R, attPerm_{IHA}, IHA)$ ; **// e.g.**

$item := selectItemFromSubset(S[ ], type_j, w_j, h_j)$

$addItemToInstance(item)$ ;

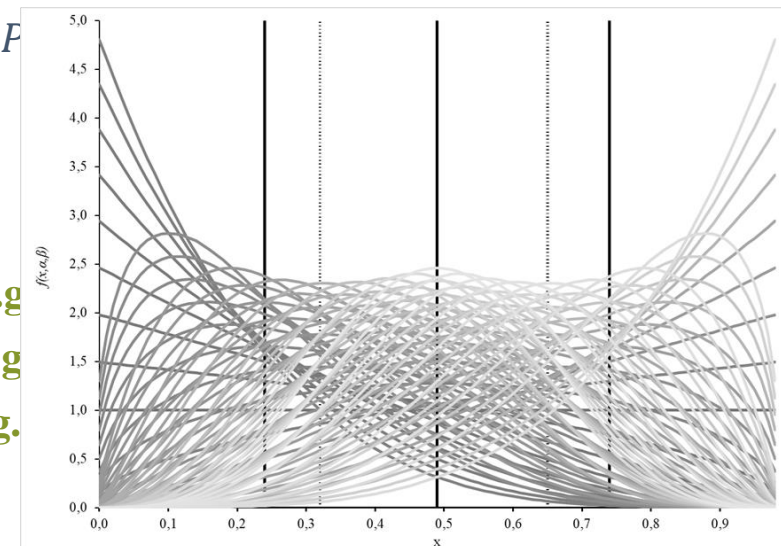
**// object width - 3**

**// item type assortment - 7**

**// item type heterogeneity - 2**

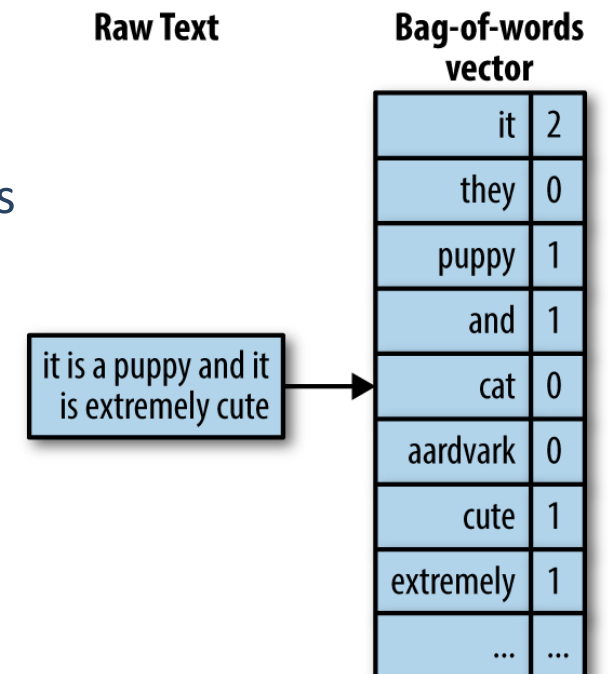
**// item width assortment - 6**

**// item height assortment - 7**



# Feature engineering

- Problem Instance Encoding: Aggregated Geometrical Representation instead of “Bag-of-Words”
- Geometrical Representation: Sum of Area (and variations), Number of vertices ...
- Advantage: more flexibility in terms of input dimension
  - Machine Learning Model can be used even for instances with new items
  - Dimension reduction methods can be used straightforward



# Feature engineering

## ■ Basic instance features

- 43 item properties (like  $h_j$ ,  $r_j^{CH} = a_j^{CH} / a_j^{MBR}$  - rectangularity of the convex hull,  $n_j^{XIA-LA} = n_j^{XIA} / n_j^E$  - rel. number of reflex interior angles, ...)
- Aggregation function for calculating instance features: *SUM*, *MED* (median), *MIN*, *MAX*, *VAR* (variance), *Q1* (first quartile), *Q3* (third quartile), *P10* (10% percentile), *P90* (90% percentile), and *SKEW* (Fisher-Pearson coefficient of skewness).

→ 430 features

## ■ Additional instance features (22):

→ 452 instance features (TIF)

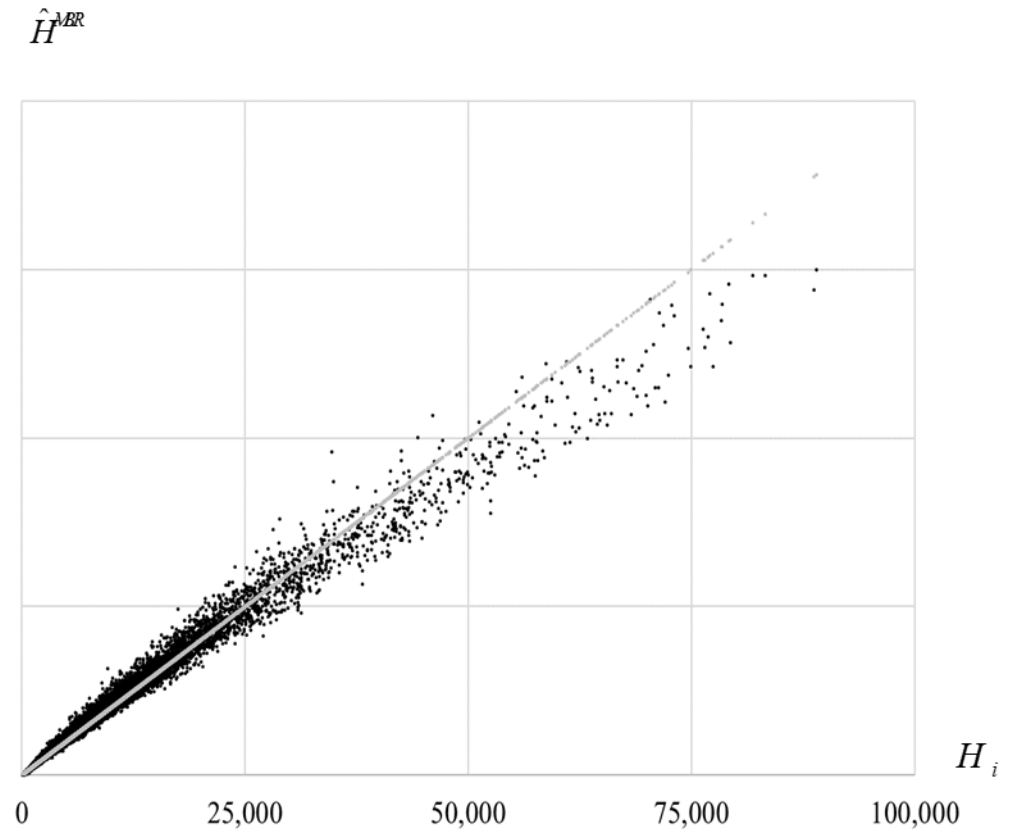
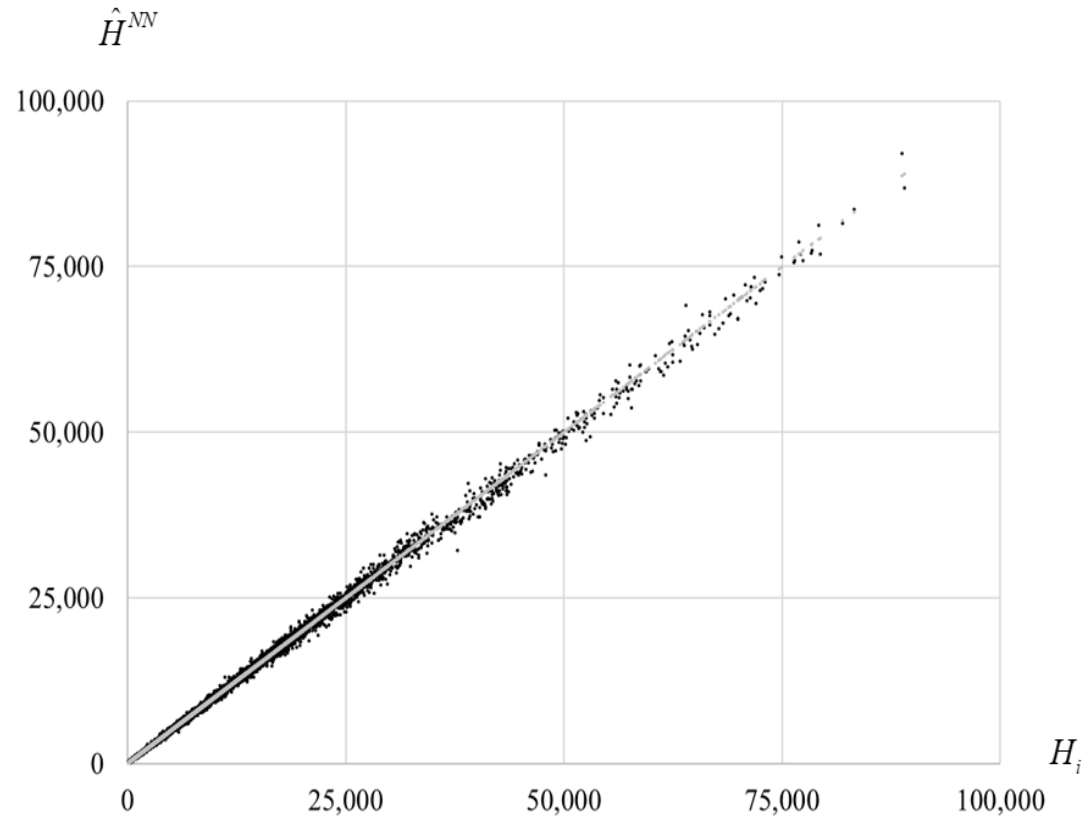
→ reduced set with 189 features (RIF) without computationally complex features

| Feature         | Description  |
|-----------------|--|
| $n$             | Total number of items  |
| $W$             | Width of the strip   |
| $\hat{H}^E$     | Predicted height based on the area of the enclosing polygon  |
| $\hat{H}^{CH}$  | Predicted height based on the area of the enclosing polygon's convex hull  |
| $\hat{H}^{MBR}$ | Predicted height based on the area of the enclosing polygon's MBR  |
| $n^D$           | Number of different item categories; two items have a different category if they are not completely identical regarding the combination of the attributes $BT \in \{CV, CA, CX\}$ , $IW \in \{S, M, L, XL\}$ , and $IH \in \{Q, HN, N\}$ . |
| $h = n^D / n$   | Heterogeneity of items   |
| $MIN^{#IpC}$    | Minimum number of items regarding all item categories  |
| $MAX^{#IpC}$    | Maximum number of items regarding all item categories  |
| $MEAN^{#IpC}$   | Mean of the number of items regarding all item categories  |
| $MED^{#IpC}$    | Median of the number of items regarding all item categories  |
| $VAR^{#IpC}$    | Variance of the number of items regarding all item categories  |
| $SKEW^{#IpC}$   | Skewness of the number of items regarding all item categories  |
| $Q1^{#IpC}$     | First quartile of the number of items regarding all item categories  |
| $Q3^{#IpC}$     | Third quartile of the number of items regarding all item categories  |
| $P10^{#IpC}$    | 10% percentile of the number of items regarding all item categories  |
| $P90^{#IpC}$    | 90% percentile of the number of items regarding all item categories  |
| $p^{LI}$        | Percentage shares of large items ( $w_j \geq 0.75 \cdot W$ ); *  |
| $p^{SI}$        | Percentage shares of small items ( $w_j \leq 0.25 \cdot W$ ); *  |
| $p^{HR}$        | Percentage shares of items with a high rectangularity ( $r_j^E > 0.9$ ); *   |
| $p^{LR}$        | Percentage shares of items with a low rectangularity ( $r_j^E \leq 0.5$ ); *   |
| $p^{NCON}$      | Percentage shares of non-convex items (items with $n_j^{XIA} > 0$ )  |
| $p^{COMP}$      | Percentage shares of complex items (items with $n_j > 1$ )   |

# Results

| RM                               | RIF             |                   | TIF           |                   |
|----------------------------------|-----------------|-------------------|---------------|-------------------|
|                                  | RMSE            | mean CT [seconds] | RMSE          | mean CT [seconds] |
| Polynomial elastic net (PEN)     | 471.58          | 42.19             | 383.96        | 240.40            |
| Bagging regression tree with PEN | 470.84          | 3,284.89          | 383.22        | 17,082.95         |
| Neural network                   | 374.96          | 6,135.63          | <b>339.17</b> | 56,569.90         |
| SA-MBR                           | 1,230.48 [RMSE] |                   |               |                   |

# Results



# Conclusions

|                                     |     | Over-estimations | Underestimations below or equal |       |
|-------------------------------------|-----|------------------|---------------------------------|-------|
|                                     |     |                  | 5%                              | 10%   |
| SA-MBR                              |     | 62.4%            | 79.2%                           | 89.9% |
| NN with PCA(98%),<br>“DiffLab”, and | RIF | 60.8%            | 92.9%                           | 98.6% |
|                                     | TIF | 58.5%            | 93.4%                           | 98.8% |

## ■ Outlook

- Prediction Intervalls → Uncertainty Quantification, Bayesian methods
- Extension to 3D-Nesting: Capacity Checking, Batch Processing Time
- How to include the ML-model into the hierarchical problem?

Thank You For Your Attention